Vibration Analysis of Woven Fiber Glass/Epoxy Composite Plates

A Thesis Submitted In Partial Fulfillment
of the Requirements for the degree of

Master of Technology
In
Civil Engineering
(Structural Engineering)

By

Parsuram Nayak

Department of Civil Engineering

National Institute of Technology Rourkela
Rourkela-769008,
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Under The Guidance of

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NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA – 769008, ORISSA

CERTIFICATE

This is to certify that the thesis entitled, "**Vibration Analysis of Woven Fibre Glass/Epoxy Composite Plate**" submitted by **Mr. Parsuram Nayak,** in partial fulfillment of the requirement for the award of **Master of Technology** in Civil Engineering with specialization in "**Structural Engineering**" at the National Institute of Technology, Rourkela during the academic year 2006-08 in an authentic work carried out by her under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

Date: 27.05.2008

Dr. S. K. Sahu
Professor
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Date: 27.05.2008

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CONTENTS

ABSTRACT

This work presents a combined experimental and numerical study of free vibration of woven fiber Glass/Epoxy composite plates. Experimental setup and procedure of the modal testing method is described. Fabrication procedure of the plate is described. Geometrical properties are determined. Elastic parameters of the plate are determined experimentally by tensile testing of specimens. A computer program based on FEM has been developed to perform all necessary computations. The program results compared with other existing literature. The natural frequencies of 12-layered and 16-layered woven fiber Glass/Epoxy cantilevered composite plates has been determined experimentally and compared with the present program. The natural frequency and mode shape of the plate has been determined using ANSYS package. The present experimental value and program result compared with ANSYS package. The effects of varying the parameters upon the free vibration frequencies are discussed for 12-layered cantilevered and all edge clamped woven fiber Glass/epoxy composite plates by using the program.

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Chapter 1

INTRODUCTION

INTRODUCTION

A large variety of fibers are available as reinforcement for composites. The desirable characteristics of most fibers are high strength, high stiffness, and relatively low density. Glass fibers are the most commonly used ones in low to medium performance composites because of their high tensile strength and low cost.

In woven fiber, fibers are woven in both principal directions at right angles to each other. Woven glass fibers is used to achieve higher reinforcement loading and consequently, higher strength. Woven glass fiber as a weight percent of laminate may be range to 65%. Woven roving are plainly woven from roving, with higher dimensional properties and regular distribution of glass fiber with excellent bonding strength among laminates possesses higher fiber content, tensile strength, impact resistance.

The combination of different materials has been used for many thousands of years to achieve better performance requirements. There are nowadays many examples in the aeronautical and automobile industries, and yet the application of composite materials is still growing, including now areas such as nautical industries, sporting goods, civil and aerospace construction.

Composite structures have been widely used in many engineering examples in aeronautical, astronautically, and marine structures. The more common composites used are laminated composite plates which are typically made of different layers bonded together. Basically, each layer is generally orthotropic and has a different orientation of the fibers.

In addition to the advantages of high strength (as well as high stiffness) and light weight, another advantage of the laminated composite plate is the controllability of the structural properties through changing the fiber orientation angles and the number of plies and selecting proper composite materials.

In order to achieve the right combination of material properties and service performance, the dynamic behavior is the main point to be considered. From a theoretical point of view this has led to the development of numerous models of composite plates for the prediction of different parameters including free vibration. To avoid the typical problems caused by vibrations, it is important to determine: a) natural frequency of the structure; b) the modal shapes to reinforce the most flexible regions or to locate the right positions where weight should be reduced or damping should be increased and c) the damping factors.

In structural acoustics, recent work in sound transmission through laminated structures has shown that the fundamental frequency is a key parameter. The natural frequencies are sensitive to the orthotropic properties of composite plates and design-tailoring tools may help in controlling this fundamental frequency. The understanding of prediction models facilitates the development of such tools. Due to the advancement in computer aided data acquisition systems and instrumentation, Experimental Modal Analysis has become an extremely important tool in the hands of an experimentalist.

This work presents an experimental study of modal testing of two different woven fiber Glass/Epoxy cantilever composite plates. A program based on FEM is developed. The result of the program has been compared with other literature. The experimental results have been compared with that obtained from the finite element analysis. Fabrication method and elastic properties of the plate determined from tensile test method. Variation of natural frequency with different parameter is studied.

Chapter 2

REVIEW OF LITERATURE

REVIEW OF LITERATURE

Due to the requirement of high performance material in aerospace and marine structures, the prospect of future research of composite material, such as FRP (Fibre Reinforced Plastic) is very bright. Analysis of natural frequency and properties of composite plate has started from 40 years ago.

The natural frequencies and mode shapes of a number of Graphite/ Epoxy and Graphite/Epoxy-Aluminum plates and shells were experimentally determined by Crawly (1979). Natural frequency and mode shape results compared with finite element method.

Alam and Asani (1986) studied the governing equations of motion for a laminated plate consisting of an arbitrary number of fiber-reinforced composite material layers have been derived using the variation principles. Each layer has been considered to be of a specially orthotropic material with its directional elastic properties depending on the fiber orientation. A solution for simply supported rectangular plate is obtained in series summation form and the damping analysis is carried out by an application of the correspondence principle of linear viscoelasticity.

Reddy (1987) discussed different shear deformation theories as applicable to composites. The formulation and application of isoparametric plate bending element, in general, have been extensively discussed by Zienkiewicz (1989).

Narita and Leissa (1991) presented an analytical approach and accurate numerical results for the free vibration of cantilevered, symmetrically laminated rectangular plates. The natural frequencies are calculated for a wide range of parameters: e.g., composite material constants, fiber angles and stacking sequences.

Qatu and Leissa (1991) analyzed free vibrations of thin cantilevered laminated plates and shallow shells by Ritz method. Convergence studies are made for spherical circular cylindrical, hyperbolic, paraboloidal shallow shells and for plates. Results are compared with experimental value and FEM. The effect of various parameters (material number of layers, fiber orientation, curvature) upon the frequencies is studied.

Rosce and Lu (1993) have determined the vibrational characteristics of a glass reinforced composite cylindrical shell experimentally and evaluated. An impedance test was conducted to study the effectiveness of damping and vibration related properties of the composite.

Soares, Pedersen and Araujo (1993) described an indirect identification technique to predict the mechanical properties of composites which makes use of eigen frequencies, experimental analysis of a composite plate specimen, corresponding numerical eigen value analysis and optimization techniques.

Liu and Huang (1994) studied to solve free vibration problems for thick cantilever laminated plates with or without a step-change of thickness in the chord-wise direction by combination of three dimensional FEA and transfer matrix method. Both symmetric and non-symmetric, isotropic and laminated plates are considered. For laminated plates, natural frequency with different fiber orientations is studied.

Free vibration analysis of symmetrically laminated, rectangular plates with clamped boundary conditions is studied using the hierarchical finite element method by Han and Petyt (1995). The frequencies produced using the hierarchical finite element method closely with results in the published literature using a Rayleigh-Ritz type of analysis.

Linear vibration analysis of laminated rectangular plates has been reported by Han and Petyt (1996), who describe the free and forced vibration analysis of symmetrically laminated rectangular plates with clamped boundary condition using hierarchical finite element techniques.

A Theoretical model to predict the response of laminated composites is developed by Mobasher (1996). The micromechanical model simulates the mechanical response of a multi layer composite laminate under uniaxial, biaxial, and flexural loading modes. The stacking sequence is utilized to obtain the overall stiffness matrix for each lamina.

Rao and Ganeshan (1997) have investigated the harmonic response of tapered composite beams by using a finite element model. The Poisson's effect is incorporated in the formulation of the beam constitutive equations.

A procedure for determining the sensitivities of the eigen values and eigenvectors of damped vibratory systems with distinct eigen values is presented by Lee, Kim and Jung (1998). The eigen pair derivatives of the structural and mechanical damped systems can be obtained by solving algebraic equations. The finite element model of a cantilever plate is considered, and also a 7-DOF half-car model in the case of damped system.

Stanbridge and Ewins (1999) described a number of vibration mode-shape measurement techniques are described in which the measurement point of a laser doppler vibrometer (LDV) is continuously scanned over the surface of a sinusoidally excited structure.

A combined experimental and numerical study of the free vibration of composite GFRP plates has been carried out by Chakraborty, Mukhopadhyay and Mohanty (2000). Modal testing has been conducted using impact excitation to determine the respective frequency response functions. FEM results, NISA package results compared with experimental results.

A procedure to estimate the dynamic damped behavior of fiber reinforced composite cantilever beams in flexural vibrations is given by Tita, Carvalho and Lirani (2001). A set of experimental dynamic tests were carried out in order to investigate the natural frequencies and modal shapes. Damping factors are evaluated by the program.

Matsunaga (2003) has analyzed natural frequencies and buckling stresses of laminated composite circular arches subjected to initial axial stress by taking into account the complete effects of transverse shear and normal stresses and rotary inertia.

Khalili, Malekzadeh and Mittal (2004) have presented a new analytical method is developed to analyze the response of laminated composite plates subjected to static and dynamic loading. The modal forms are presented in terms of double Fourier series. The results from the present analysis are compared with those obtained from the FEM code NISA.

The natural frequencies and corresponding vibration modes of a cantilever sandwich beam with a soft polymer foam core are predicted using the higher-order theory for sandwich panels (HSAPT), a two-dimensional finite element analysis, and classical sandwich theory by Sokolinsky, Bremen, Lavoie and Nutt (2004).

The small amplitude vibration characteristics of thermally stressed laminated composite skew plates are studied by Singha, Ramanchandra and Bandyopadhyay (2006) using a shear deformable finite element. The first three natural frequencies are studied in the pre- and post buckled states. Limited parametric study has been carried out to study the influences of fiber orientation, skew angle, and boundary condition on the vibration characteristics of thermally stressed composite plate.

Berthelot and Sefrani (2006) investigates the damping of unidirectional glass fiber composites with a single or two interleaved viscoelastic layers. The experimental damping characteristics are derived from flexural vibrations of cantilever beams as a function of the fiber orientation.

Composite laminate structures can be designed for specific purposes by optimizing the number of plies and the ply orientations Woodcock, Bhat and Stiharu (2007).

Laila (2008) has presented aeroelastic characteristics of a cantilevered composite wing, idealized as a composite flat plate laminate. The composite laminate was made from woven glass fibers with epoxy matrix. The elastic and dynamic properties of the laminate were determined experimentally for aeroelastic calculations.

Chapter 3

THEORY AND FORMULATION

THEORY AND FORMULATION

3.1 FREE VIBRATION

By free vibration we mean the motion of a structure without any dynamic equation external forces or support motion. The motion of the linear SDF systems without damping specializes to

$$m\frac{d^2u}{dt^2} + ku = 0$$

Free vibration is initiated by disturbing the system from its static equilibrium position. By imparting the mass some displacement u (0) and velocity $\dot{u}(0)$ at time zero, defined as the instant the motion is initiated:

$$u = u(0), \quad \dot{u} = \dot{u}(0)$$

So, solution to the equation is obtained by standard methods:

$$u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t$$

Where natural circular frequency of vibration in unit radians per second= $\omega_n = \sqrt{\frac{k}{m}}$

The time required for the undamped system to complete one cycle of free vibration is the natural period of vibration of the system.

$$T_n = \frac{2\pi}{\omega_n}$$

Natural cyclic frequency of vibration is denoted by $f_n = \frac{1}{T_n}$, unit in Hz (cycles per Second).

3.2 MODE SHAPE

We introduce the eigen value problem whose solution gives the natural frequencies and modes of a system. The free vibration undamped system in one of its natural vibration modes can be described by

$$u(t) = q_n(t)\phi_n$$

Where, Φ_n does not vary with time.

The time variation of the displacements is described by the simple harmonic function

$$q_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t$$

A_n, B_n are constants of integration.

Combining above two equations we have

$$u(t) = \phi_n (A_n \cos\omega_n t + B_n \sin\omega_n t)$$

Putting in equation of undamped free vibration, we have

$$[-\omega_n^2 m\phi_n + k\phi_n]q_n(t) = 0$$

Either, $q_n(t) = 0$, $\Rightarrow u(t) = 0$, trivial solution

Or, $k\phi_n = \omega_n^2 m\phi_n$

This is called matrix eigen value problem.

This equation can be written as

$$[k - \omega_n^2 m]\phi_n = 0$$

A set of 'n' homogeneous algebraic equation is for that 'n' no of element. This set has always the trivial solution $\phi_n = 0$, it implies no motion.

The nontrivial solution is

det$[k - \omega_n^2 m]$ =0, This is called frequency equation.

It gives N roots in ω_n^2 determine N natural frequencies. The roots are called eigen value or normal values. Corresponding to the N natural vibration frequencies ω_n of an N-DOF System, there are N independent vector ϕ_n which are called natural mode shapes of vibration, eigenvector, normal modes.

3.3 FINITE ELEMENT ANALYSIS

The equations of equilibrium of a discretised elastic structure undergoing small deformations can be expressed as

$$[M]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{F(t)\} \qquad (1)$$

For free undamped vibration, the equation reduces to

$$[M]\{\ddot{u}\} + [k]\{u\} = \{0\} \qquad (2)$$

If modal co-ordinates are employed the equation becomes

$$\{[k]-\omega^2[M]\}\{\phi_n\}=\{0\} \qquad (3)$$

There are various methods of finding the natural frequencies ω_i and modal vectors $\{\phi_n\}_i$ once the system mass *[M]* and stiffness matrices *[K]* are formulated. Here an eight noded isoparametric plate bending element has been chosen to discretise the plate. The necessary constitutive relationships have also been formed. The element is capable of incorporating transverse shear deformation through the implementation of First Order Shear Deformation Theory (3) as applicable to composite (4).

The element stiffness matrix can be expressed as

$$[K]_e=\int_{-1}^{+1}\int_{-1}^{+1}[B]^T[D][B]|J|\,d\xi\,d\eta \qquad (4)$$

The Gaussian Quadrature formula is used for numerical integration. Reduced integration technique has been employed in order to avoid shear locking [5]. Similarly the consistent element mass matrix is generated using,

$$[M]_e=\int_{-1}^{+1}\int_{-1}^{+1}[N]^T[m][N]|J|\,d\xi\,d\eta \qquad (5)$$

Effect of rotary inertia is neglected.

Chapter 4

EXPERIMENTAL PROGRAMME

EXPERIMENTAL PROGRAMME

4.1 GEOMETRICAL PROPERTY

In order to respect the assumption of classical theory of bending of thin plates with small deflections, keep the thickness of the plate smaller than $1/5^{th}$ of the largest dimension of the plate. The thickness of the test plates was even more reduced up to $1/10^{th}$ of the largest dimension of the plate, in order to keep resonant frequencies of the test structure as low as possible, thus assuring good vibration measurements.

In choosing the types of specimens to construct and test, woven fibered Glass/Epoxy composite plates were taken. Two woven fiber Glass/Epoxy composite plates were taken. It was prepared to cast as cantilever one by sand mortar mixture. The length of cantilever plate was 14cm for each case. The average thickness of all specimens was measured by a screw gauge having a least count of 0.01mm. The Length, breadth, thickness of cantilever plate is shown on below.

Table 4.1: Length (a), breadth (b), Thickness (h) of the Glass/Epoxy Plate

Plate no	Total length (cm)	Cantilevered length (cm)	Breadth (cm)	Thickness (cm)
1 (GFRP)	19.5	14	5.7	0.4
2(GFRP)	19.7	14	6.4	0.7

4.2 FABRICATION METHOD

To meet the wide range of needs which may be required in fabricating composites, the industry has evolved oven a dozen separate manufacturing processes as well as a number of hybrid processes. Each of these processes offers advantages and specific benefits which may apply to the fabricating of composites. Hand lay-up and spray-up are two basic molding processes. The hand lay-up process is the oldest, simplest, and most labour intense fabrication method. The process is most common in FRP marine construction. In hand lay-up method liquid resin is placed along with reinforcement (woven glass fiber) against finished surface of an open mould. Chemical reactions in the resin harden the material to a strong, light weight product. The resin serves as the matrix for the reinforcing glass fibers, much as concrete acts as the matrix for steel reinforcing rods. The percentage of fiber and matrix was 50:50 in weight.

Contact moulding in an open mould by hand lay-up was used to combine plies of WR in the prescribed sequence. A flat plywood rigid platform was selected. A plastic sheet was kept on the plywood platform and a thin film of polyvinyl alcohol was applied as a releasing agent by use of spray gun. Laminating starts with the application of a gel coat (epoxy and hardener) deposited on the mould by brush, whose main purpose was to provide a smooth external surface and to protect the fibers from direct exposure to the environment. Ply was cut from roll of woven roving. Layers of reinforcement were placed on the mould at top of the gel coat and gel coat was applied again by brush. Any air which may be entrapped was removed using serrated steel rollers. The process of hand lay-up was the continuation of the above process before the gel coat had fully hardened. Again, a plastic sheet was covered the top of plate by applying polyvinyl alcohol inside the sheet as releasing agent. Then, a heavy flat metal rigid platform was kept top of the plate for compressing purpose. The plates were left for a minimum of 48 hours before being transported and cut to exact shape for testing. The following constituent materials were used for fabricating the plate:

1. E-glass woven roving as reinforcement
2. Epoxy as resin
3. Hardener as diamine (catalyst)
4. Polyvinyl alcohol as a releasing agent

4.3 DETERMINATION OF MATERIAL CONSTANTS

The characteristics of woven fiber Glass/Epoxy composite plate which can be defined completely by four material constants: E_1, E_2, G_{12}, and υ_{12} where the suffixes 1 and

2 indicate principal material directions. For material characterization of composites, laminate having 12 layers was manufactured to evaluate the material constants.

The constants are determined experimentally by performing unidirectional tensile tests on specimens cut in longitudinal and transverse directions, and at 45° to the longitudinal direction, as described in ASTM standard: D 638-08 and D 3039/D 3039M - 2006. A thin flat strip of specimen having a constant rectangular cross section was prepared in all cases. The dimension of the specimen was taken as below:

Table 4.2: Size of the specimen for tensile test

Length(mm)	**Width(mm)**	**Thickness(mm)**
165	13	4

The specimens were cut from the plates themselves by diamond cutter or by hex saw. After cutting in the hex saw, it was polished in the polishing machine. At least three replicate sample specimens were tested and mean values adopted.

Coupons were machined carefully to minimize any residual stresses after they were cut from the plate and the minor variations in dimensions of different specimens are carefully measured. For measuring the Young's modulus, the specimen is loaded in INSTRON 1195 universal testing machine monotonically to failure with a recommended rate of extension (rate of loading) of 5 mm/minute. Specimens were fixed in the upper jaw first and then gripped in the movable jaw (lower jaw). Gripping of the specimen should be as much as possible to prevent the slippage. Here, it was taken as 50mm in each side. Initially strain was kept at zero. The load, as well as the extension, was recorded digitally with the help of a load cell and an extensometer respectively. From these data, engineering stress vs. strain curve was plotted; the initial slope of which gives the Young's modulus. The ratio of transverse to longitudinal strain directly gives the Poisson's ratio by using two strain gauges in longitudinal and transverse direction. But, here Poisson’s ratio is taken as 0.17 from Bureau Veritus (1979).

The shear modulus was determined using the following formula from Jones (1975) as:

$$G_{12} = \frac{1}{\frac{4}{E_{45}} - \frac{1}{E_1} - \frac{1}{E_2} + \frac{2\vartheta_{12}}{E_1}}$$

The values of material constants finally obtained experimentally are presented in table below.

The material constants for 16-layered woven fiber Glass/Epoxy composite plate is taken same as 12-layered plate.

Table 4.3: Material properties of the plate as used in the program

Plate	Lay-up	n	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	ν_{12}	ρ(kg/m^3)
1	WR	12	7.57	7.57	2.8	0.17	1914
2	WR	16	7.57	7.57	2.8	0.17	1780

1. WR :- Woven Roving
2. n :- Number of layers
3. E_1, E_2 :- Elastic modulus in longitudinal direction(1) and transverse direction(2)
4. E_{45} :- Tensile modulus obtained in 45° tensile test = 7.04 GPa
5. G_{12} :- In-plane shear modulus
6. ν_{12} :- Poisson's ratio
7. ρ :- Density

Fig 4.1: Tensile test of woven fiber Glass/Epoxy composite specimen in INSTRON 1195 UTM

Fig 4.2: Failure pattern of woven fiber Glass/Epoxy composite specimen

4.4 INSTRUMENT USED

Fig4.3: Modal Impact Hammer (type 2302-5)

Fig 4.4: Accelerometer (Bruel & kjaer type 4507)

Fig 4.5: Bruel & Kajer FFT (spectrum) Analyzer

Fig 4.6: Vibration testing of cantilevered GFRP

4.5 DESCRIPTION OF TEST SPECIMEN

In order to respect the assumptions of classical theory of bending of thin plates with small deflections, keep the thickness of plate smaller than 1/5th the largest dimension of the plate. The thickness of test plates was even more reduced up to 1/10th of the largest dimension of the plate, in order to keep resonant frequencies of the test structure as low as possible, thus assuring good vibration measurement.

The Glass/Epoxy plates were prepared to cast as cantilevered one. Two (12-layered and 16-layered) plates were properly inserted to the concrete inside the moulds and compacted through the vibrators. After seven days of curing the concrete was used for testing.

4.6 SETUP AND PROCEDURE

The connections of FFT analyzer, laptop, transducers, modal hammer, and cables to the system were done as per the guidance manual. The pulse lab shop version-9.0 software key was inserted to the port of laptop. The plate was excited in a selected point by means of a small impact hammer (Model 2302-5), preferably at the fixed end. The input signals captured by a force transducer, fixed on the hammer. The resulting vibrations of the plate in a select point are measured by an accelerometer. The accelerometer (B&K, Type 4507) was mounted on the plate to the free end by means of bees wax. The signal was then subsequently input to the second channel of the analyzer, where its frequency spectrum was also obtained. The response point was kept fixed at a particular point and the location of excitation was varied throughout the plate.

Both input and output signals are investigated by means of spectrum-analyzer (Bruel & kjaer) and resulting frequency response functions are transmitted to a computer for modal parameter extraction.

The output from the analyzer was displayed on the analyzer screen by using pulse software. Various forms of Frequency Response Functions (FRF) are directly measured.

However, the present work represents only the natural frequencies and mode shape of plates. The spectrum analyzer provided facilities to record all the data displayed on the screen to a personal computer's hard disk or laptop and the necessary software.

Normally in order to determine the natural frequencies of a system, recording the response spectrum for an excitation, where the excitation level is constant over the frequency band under consideration will suffice. However, it was observed, from the auto-spectrum of the excitation force, that it was not possible to maintain such uniform excitation in case of composite plates. So, test should be within linear range.

The hammer excitation method is fast and simple method. A sharp impact pulse corresponds to a large frequency domain. Unfortunately, since the energy of the force pulse is limited, the method has poor signal to noise characteristics, but the noise can be minimized by using an adequate weighting function. Nevertheless, the composite plates showed very rapidly to have frequencies above 2000Hz, which are difficult to excite with enough energy by means of a hammer.

Chapter 5

RESULTS AND DISCUSSION

RESULTS AND DISCUSSION

5.1 COMPARISION OF PROGRAM RESULT

One FEM based program is developed to calculate the natural frequencies of the cantilevered plates. Natural frequencies are computed from the program and compared with some data in existing literature. In Table 5.1, the natural frequencies are compared for a Graphite/Epoxy cantilevered square plate, Crawly (1979). In Table 5.2, it is compared for a Graphite/Epoxy cantilevered rectangular plate, Crawly (1979).

Natural frequencies are calculated from the present program and compared with Chakraborty (2000) in Table 5.3 and Table5.4 for all edge clamped woven Glass/Epoxy composite plate. In Table 5.3 stacking sequence is (0/0/0/0/0) and (0/45/0/45/0) for Table 5.4.

It is shown from the Table 5.1, 5.2, 5.3, 5.4 that the natural frequencies from present program agree well with those obtained in literature Crawly (1979) and Chakraborty (2000).

Table 5.1: Comparison of natural frequencies (Hz) from present program with Table 3 in Crawly (1979) for a cantilevered square plate with following data;

8-ply Graphite/Epoxy plate, Length=76mm, Breadth=76mm, Thickness=1.04mm, E_1=128GPa, E_2=11GPa, ν_{12}=0.25, G_{12}=4.48GPa, G_{13}=1.53GPa, ρ=1500kg/m^3

Laminate	Mode No	Observed Frequency(Hz)	Calculated Frequency(Hz)	Present Program
$[0_2/\pm30]_s$	1	234.2	261.9	261.2338
	2	362	363.5	361.4863
	3	728.3	761.8	754.6239
	4	1449	1662	1590.0505

Table 5.2: Comparison of natural frequencies (Hz) from present program with Table4 in Crawly (1979) for a cantilevered rectangular plate with following data;

8-ply Graphite/Epoxy plate, Length=152mm, Breadth=76mm, Thickness=1.04mm, E_1=128GPa, E_2=11GPa, ν_{12}=0.25, G_{12}=4.48GPa, G_{13}=1.53GPa, ρ=1500kg/m^3

Laminate	Mode No	Observed Frequency(Hz)	Calculated Frequency(Hz)	Present Program
$[0/\pm45/90]_s$	1	48.6	55.58	55.4511
	2	169	175.4	174.1699
	3	303	345.3	344.1610
	4	554	591.8	587.9175

Table 5.3: Comparison of natural frequencies (Hz) from present program with Table4 in Chakraborty (2000) for all edges clamped plate (plate A) with following data;

Length=0.333m, Breadth=0.333m, Thickness=0.0027m, E_{11}=E_{22}=14.4GPa, G_{12}=2.68GPa, ν_{12}=0.17, ρ=1529kg/m^3

Plate (A)	Mode No	Experimental	FEM	Present program
Woven Roving, (0/0/0/0/0), No of Plies=5	1	116.0	118.49	118.5823
	2	232.0	243.14	243.6441
	3	320.0	344.39	346.4656
	4	412.0	443.43	_

Table 5.4: Comparison of natural frequencies (Hz) from present program with Table4 in Chakraborty (2000) for all edges clamped plate (Plate C) with following data;

Length=0.333m, Breadth=0.333m, Thickness=0.00305m, E_{11}=E_{22}=11.55GPa, G_{12}=2.68GPa, ν_{12}=0.17, ρ=1611kg/m^3

Plate (C)	Mode No	Experimental	FEM	Present program
Woven Roving,	1	120	117.81	117.9026
(0/45/0/45/0),	2	248	240.87	241.3491
No of Plies=5	3	360	348.56	350.5101
	4	456	435.07	_

5.2 PULSE REPORT

5.2.1PULSE REPORT:(For 12-layer Woven Fiber Glass/Epoxy Cantilever Composite Plate)

Fig 5.1: Frequency ~ Response

In X-axis: Frequency in Hz

In Y-axis: Acceleration in m/s^2

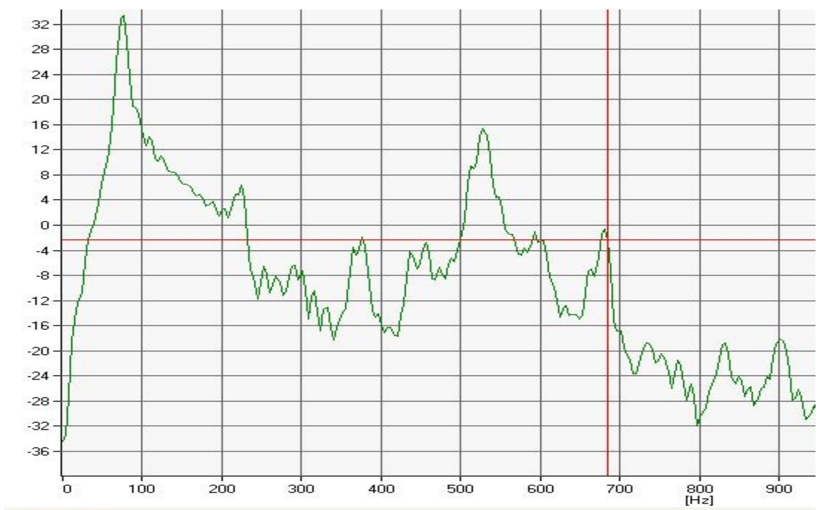


Fig 5.2: Frequency ~ Force

In X-axis: Frequency in Hz

In Y-axis: Force in Newton

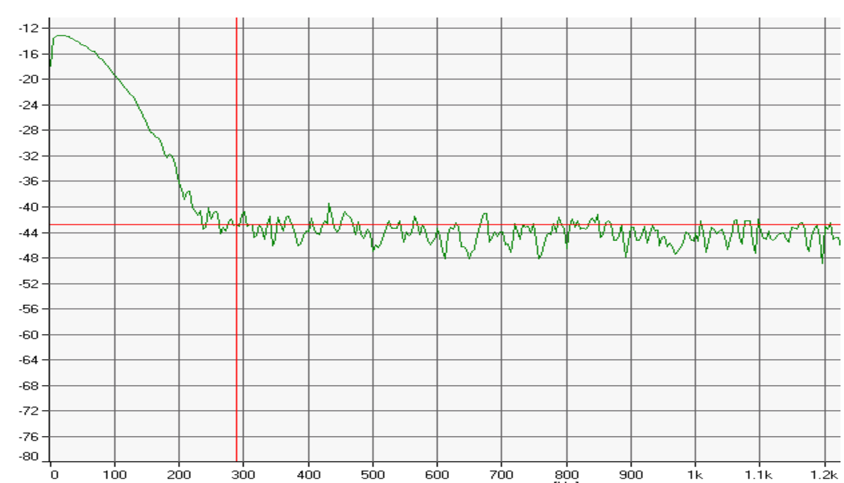


Fig 5.3: Time ~ Force

In X-axis: Time in Sec

In Y-axis: Force in Newton

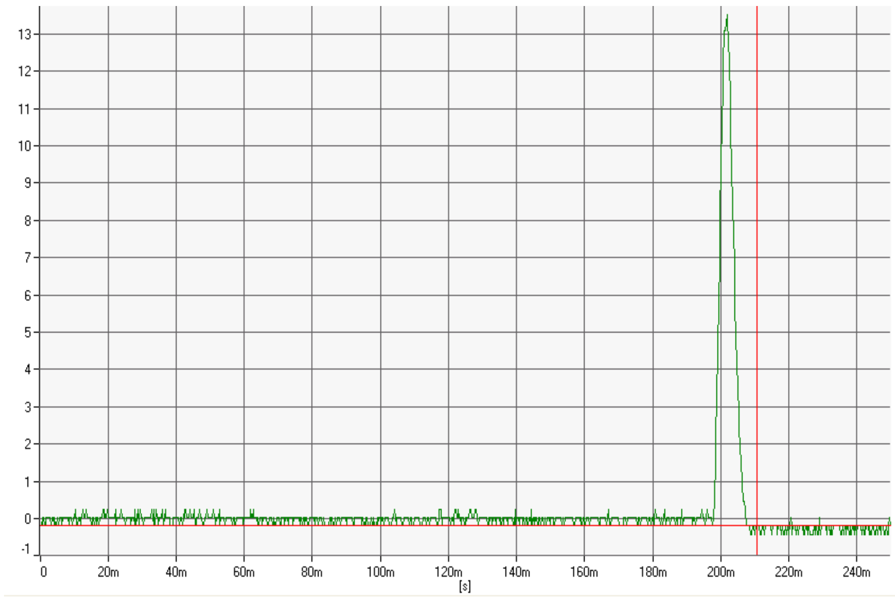
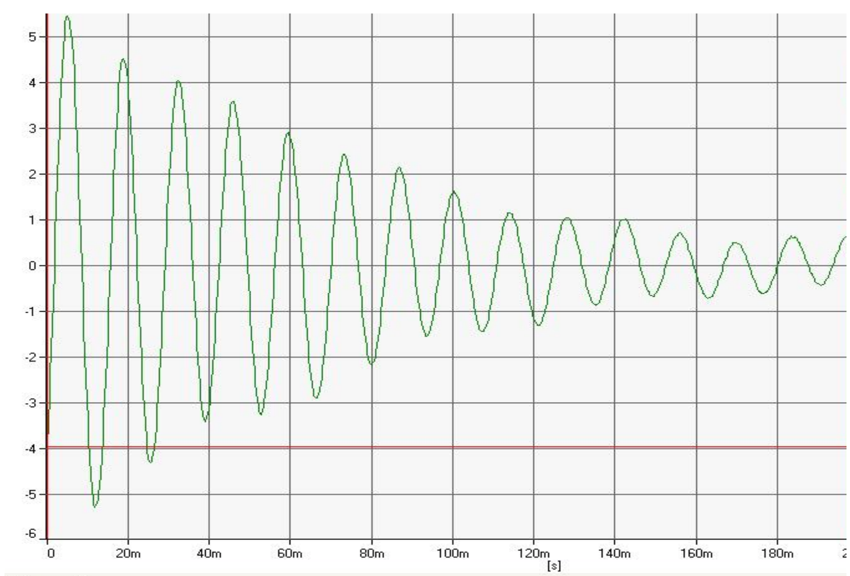


Fig 5.4: Time ~ Response

In X-axis: Time in Sec

In Y-axis: Acceleration in m/s^2



5.2.2 PULSE REPORT:(For 16-layer Woven Fiber Glass/Epoxy Cantilever Composite Plate)

Fig 5.5: Frequency ~ Response

In X-axis: Frequency in Hz

In Y-axis: Acceleration in m/s^2

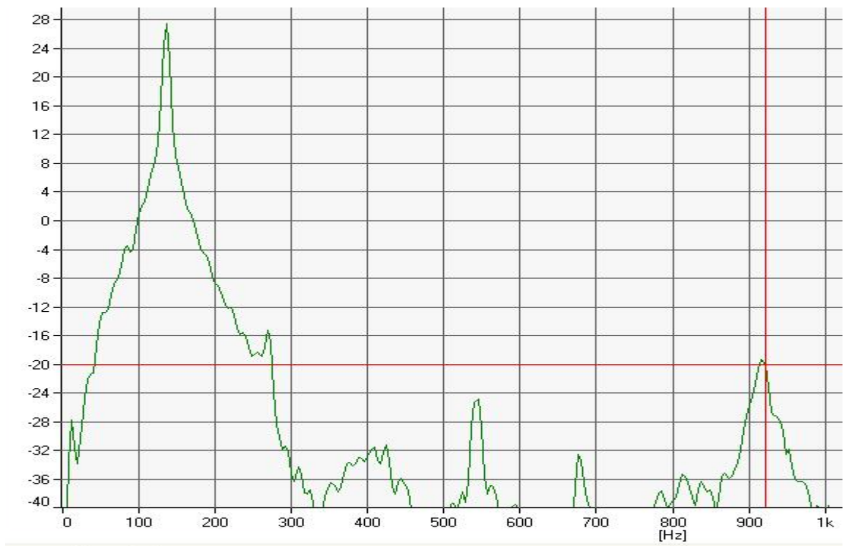
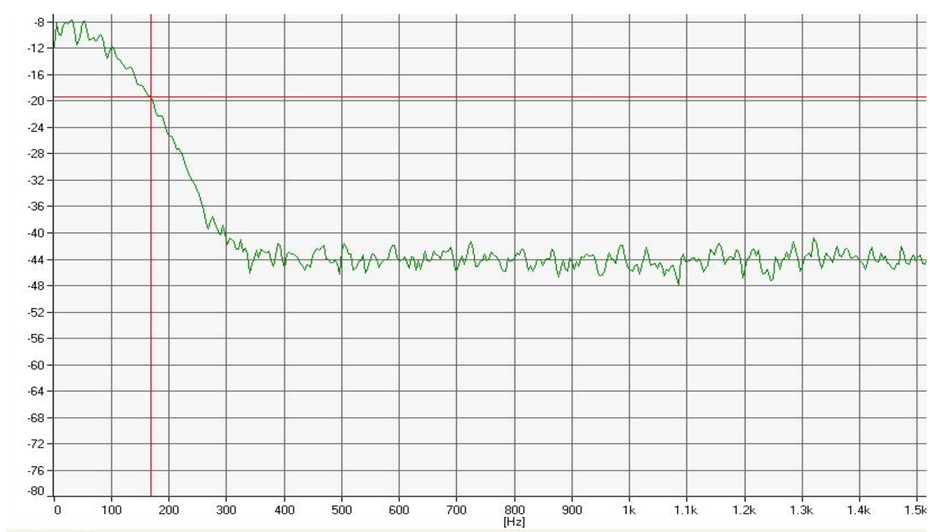


Fig 5.6: Frequency ~ Force

In X-axis: Frequency in Hz

In Y-axis: Force in Newton



5.3 RESULT OF MODAL TESTING

Table 5.5: Comparison of experimental natural frequencies (Hz) and programming frequencies for 12-layered and 16-layered Glass/Epoxy cantilevered composite Plates

Plate No	Mode No	Experimental	Program	% Error of Experiment with program
1(12-layer)	1	76	65.9411	-15.25
	2	360 - 372	331.9557	-8.45 to -12.06
	3	440 - 454	411.1787	-7.01 to -10.41
	6	836	834.2902	-0.2
2(16-layer)	1	136	119.5090	-13.80
	2	550	533.4217	-3.11
	3	680	739.1508	+8.00
	4	920	947.3833	+2.89

Natural frequencies of woven fiber Glass/Epoxy cantilevered composite plates were measured by modal hammer testing method. Natural frequencies of 12-layered and 16-layered woven fiber Glass/Epoxy cantilevered composite plates were found out experimentally. The program developed by FEM has used to measure the frequencies of the two plates. The results found from the experiment and program is presented in Table 5.5. The experimental and programming values were compared. Percentage error of experimental value with programming value was calculated.

In experimental result, natural mode of frequency sometimes varies within a range as shown in Table 5.5. It shows that an approximate agreement with the FEM based program. Percentage error for the 12-layered plate is within 16% and 14% for 16-layered plate. As the mode no increases, the percentage error between experimental value and programming value decreases.

Un-damped natural frequency is considered in the program and damping was present in the system. So, the natural frequency from the experiment should less than the actual value. But the difference between both the results is reasonable. The reasons are :

The standard size of the specimen is dog-boned shape. Since I have taken rectangular pieces of specimen, elastic modulus may decrease than the exact value. So, natural frequency can be decreased.

There may be variation of elastic properties of the plate, as the sample cut from the plate was different from the plate used in the case vibration testing. Tensile properties may vary with specimen preparation and with speed and environment of testing.

Present specimens couldn't aligned in the centre of the jaw, because there is a diamond shaped hole where slippage was occurred. Specimens were fixed one of the side of the jaw. So, there may be a chance of decrease of elastic modulus (Young's modulus).

Variations in the thickness of test specimens produce variations in the surface-volume ratios of such specimens, and that may influence the test results. Reducing the cross-sectional area of the specimens may also be effective.

The program result for 16-layer may change, because the elastic properties are taken same as for the 12-layered plate. Tensile test was done for the 12-layered plate. The thickness of the 16-layered plate was 7 mm. So, it was not tested in the INSTRON machine.

Table No 5.6: Comparison of Experiment Result With ANSYS

Plate No	Mode No	Experimental	ANSYS	% Error of Experiment with ANSYS
1(12-layer)	1	76	66.091	-14.99
	2	360 - 372	335.64	-7.26 to -10.83
	3	440 - 454	416.46	-5.65 to -9.01
	4	836	836.37	+0.04
	5		1077	
2(16-layer)	1	136	119.81	-13.51
	2	550	542.21	-1.44
	3	680	749.10	+9.22
	4	920	950.72	+3.23
	5		1744.1	

The natural frequency from ANSYS with experiment results is compared. The program value is closely agree with the ANSYS value. Percentage error of experimental value and ANSYS value is within 15%. The natural frequency decreases with increase of mode no. Mode shape of the 12 –layered Glass/Epoxy composite plate is drawn by ANSYS which is shown below.

Fig 5.7: Mode No 1 (66.091 H**z)**

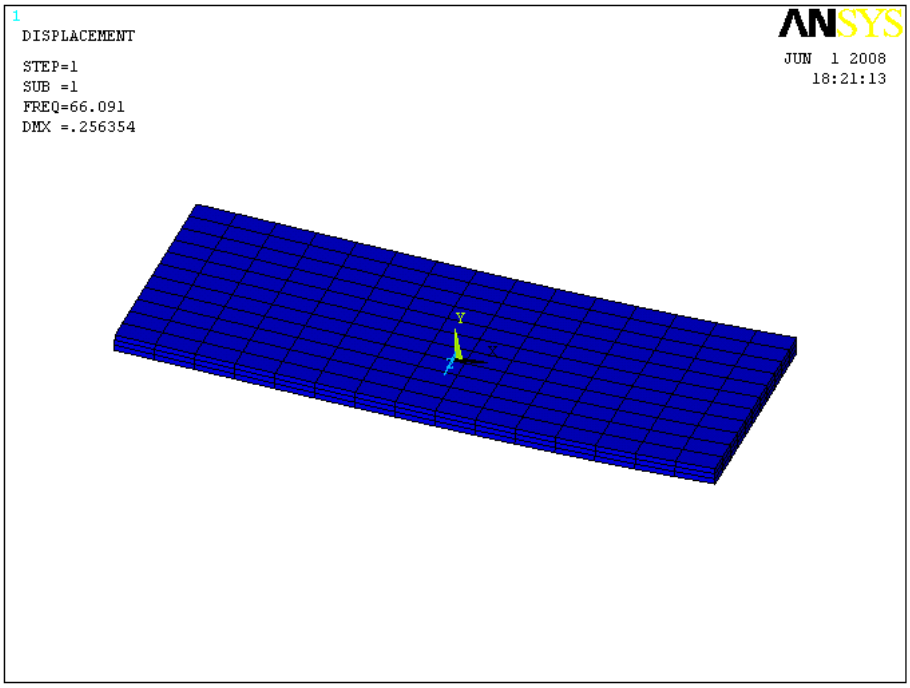


Fig 5.8: Mode No 2 (335.64 Hz)

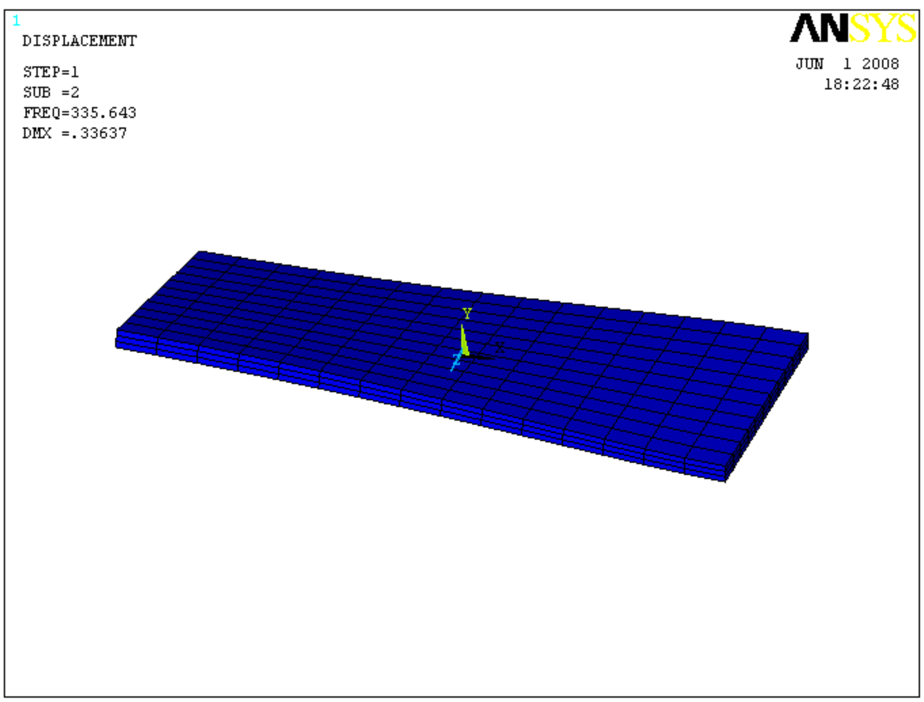


Fig 5.9: Mode No 3 (416.46 Hz)

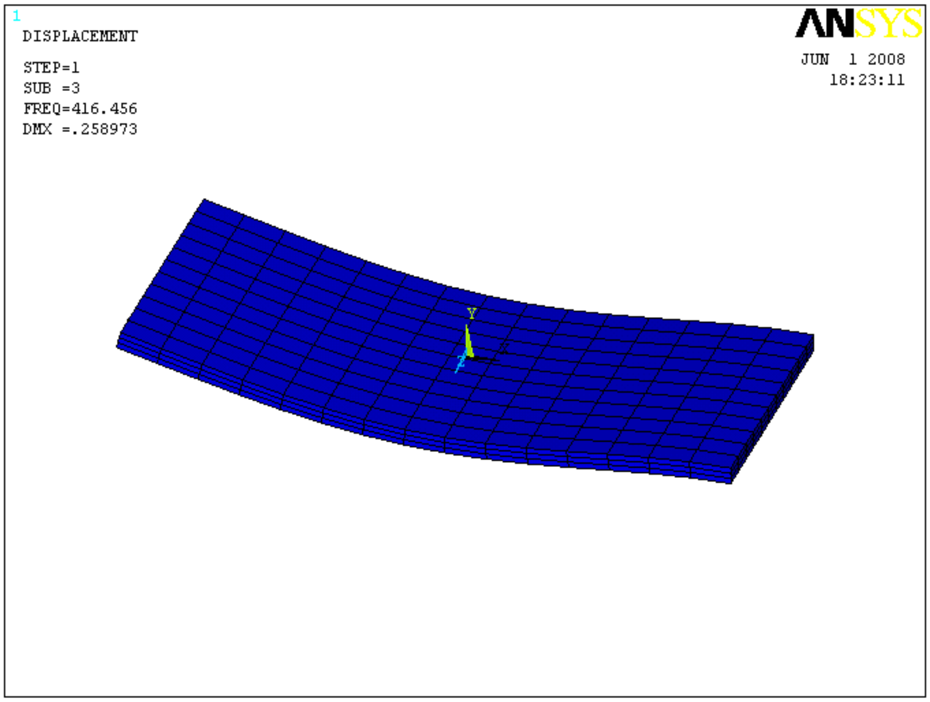


Fig 5.10: Mode No 4 (836.37 Hz)

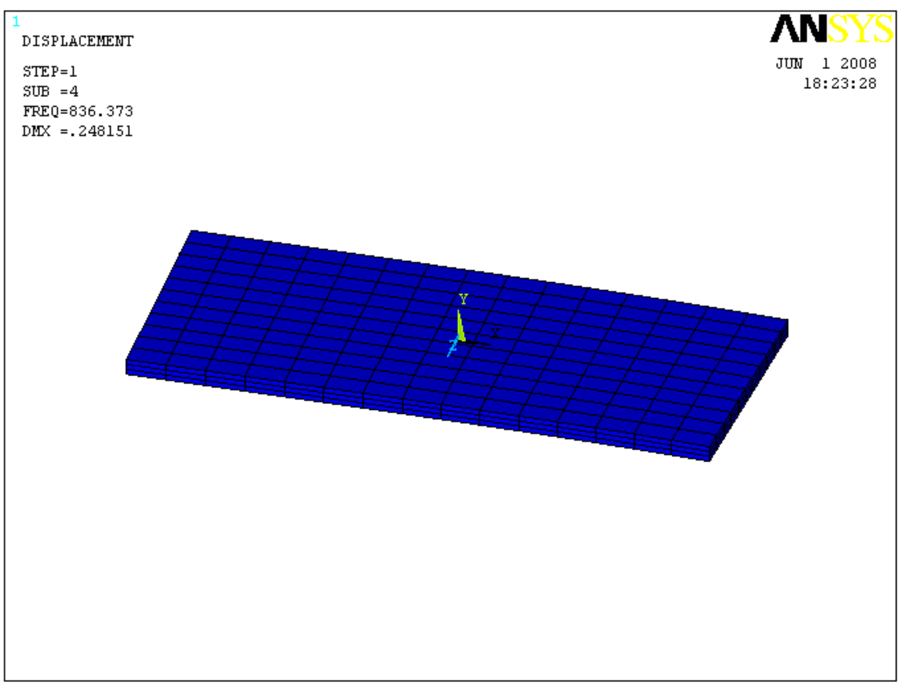
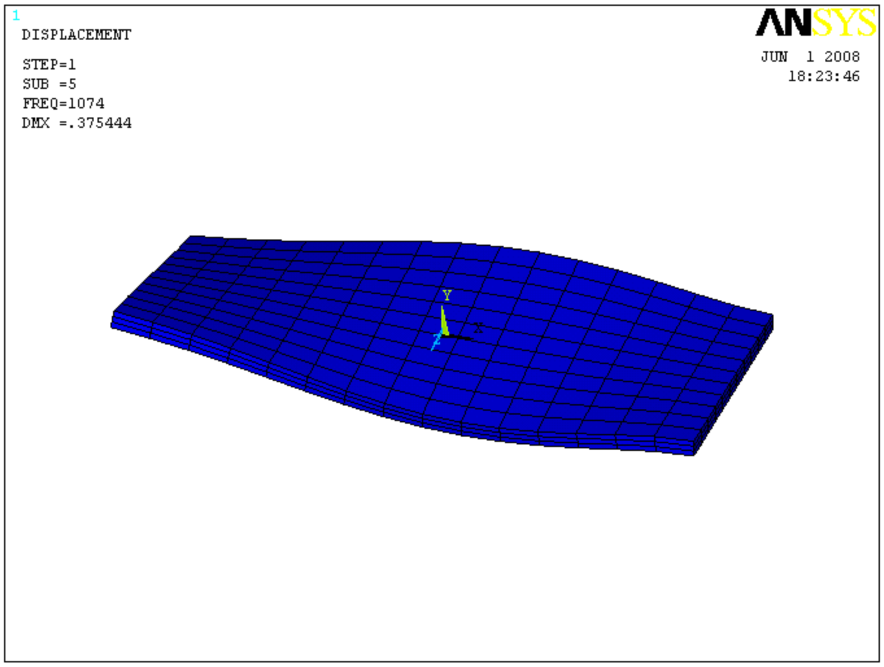


Fig 5.11: Mode No 5 (1074 Hz)



5.4 PARAMETRIC STUDY OF CANTILEVER PLATE

Table 5.7: Variation of natural frequencies with ‘a/b’ ratio for a 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, b/h=10, b=breadth, h=thickness, $E_1=E_2=10.0$GPa, $G_{12}=G_{13}=2.5$Gpa, $\nu_{12}=0.20$, $\rho=1914 kg/m^3$

a/b ratio	Mode No 1	Mode No 2	Mode No 3	Mode No 4
0.5	512.7578	683.8647	1258.1888	1798.3853
1.0	264.1025	538.9350	1523.8761	1537.6521
2.0	132.6649	464.0183	816.1814	1086.0683
3.0	89.0190	438.4614	553.1785	794.9344

Table 5.8: Variation of natural frequencies with 'b/h' ratio for a 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, a/b=2, b=breadth, h=thickness, $E_1=E_2=10.0$GPa, $G_{12}=G_{13}=2.5$Gpa, $\nu_{12}=0.20$, $\rho=1914kg/m^3$

b/h ratio	Mode No 1	Mode No 2	Mode No 3	Mode No 4
10	132.6649	464.0183	816.1814	1068.0683
25	53.2443	191.2254	332.0312	637.7885
50	26.6387	96.3300	166.4449	321.7636
100	13.3223	48.3267	83.2805	161.4719

Table 5.9: Variation of natural frequencies with '$E_{1,\,2}$' value for a 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, $G_{12}=G_{13}=2.5$Gpa, $\nu_{12}=0.20$, $\rho=1914kg/m^3$

$E_1=E_2$	Mode No 1	Mode No 2	Mode No 3	Mode No 4
5	94.0596	440.4102	582.5636	816.5224
10	132.6649	464.0183	816.1814	1068.0683
15	162.1587	482.1979	991.0707	1221.4918
20	186.9226	497.4084	1135.0361	1327.2589

Table 5.10: Variation of natural frequencies with 'ν_{12}' (poison's ratio) value for a 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, $E_1=E_2=10.0$GPa, $G_{12}=G_{13}=2.5$Gpa, $\rho=1914 kg/m^3$

ν_{12}	Mode No 1	Mode No 2	Mode No 3	Mode No 4
0.10	131.7897	463.0504	812.1529	1066.3576
0.15	132.1513	463.4868	813.8227	1067.1180
0.25	133.3382	464.6502	819.2576	1069.2114
0.30	134.1816	465.3896	823.0931	1070.5513

Table 5.11: Variation of natural frequencies with 'G_{12}' for a 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, b/h=10, b=breadth, h=thickness, a/b=2, b/h=10, $E_1=E_2=10.0$GPa, $\nu_{12}=0.20$, $\rho=1914 kg/m^3$

G_{12}	Mode No 1	Mode No 2	Mode No 3	Mode No 4
2.0	132.5272	423.5800	812.6367	1031.0002
2.5	132.6649	464.0183	816.1814	1068.0683
3.0	132.7674	500.5798	818.6301	1094.9413
4.0	132.9151	565.5407	821.8331	1131.2926

Table 5.12: Variation of natural frequencies with 'ρ' (density in kg/m^3) for 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, $E_1=E_2=10.0GPa$, $G_{12}=G_{13}=2.5Gpa$, $\nu_{12}=0.20$

ρ	Mode No 1	Mode No 2	Mode No 3	Mode No 4
1000	183.5385	641.9572	1129.1657	1477.6446
1500	149.8585	524.1558	921.9599	1206.4917
2000	129.7813	453.9323	798.4407	1044.8525
3000	105.9660	370.6341	651.9241	853.1185

Table 5.13: Variation of natural frequencies with 'θ' (angle) for a 12-layered Glass/Epoxy cantilevered composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, $E_1=E_2=10.0GPa$, $G_{12}=G_{13}=2.5Gpa$, $\nu_{12}=0.20$, $\rho=1914kg/m^3$

θ (degree)	Mode No 1	Mode No 2	Mode No 3	Mode No 4
0	132.6649	464.0183	816.1814	1068.068
15	128.1657	485.649	791.7367	1050.558
30	120.3526	532.5075	742.9811	1015.056
45	116.9127	560.1469	715.9858	997.1472
60	120.3526	532.5075	742.9811	1015.056
75	128.1657	485.649	791.7367	1050.558
90	132.6649	464.0183	816.1814	1068.068

Fig 5.12:

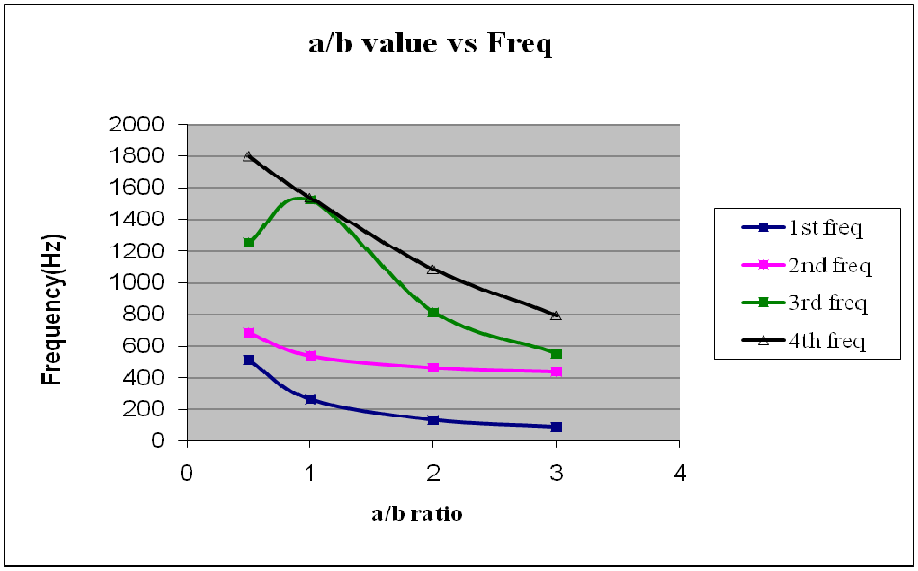


Fig 5.13:

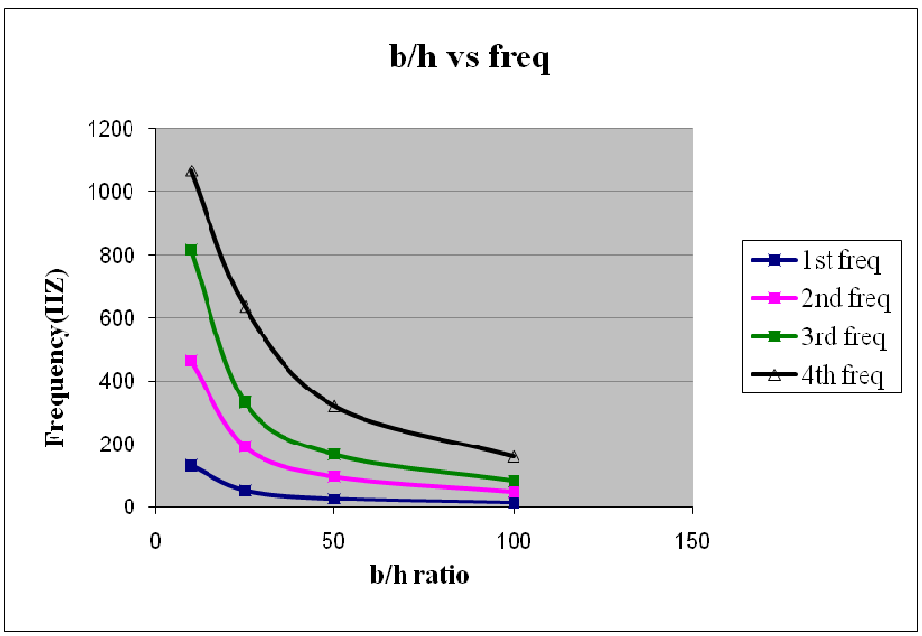


Fig 5.14:

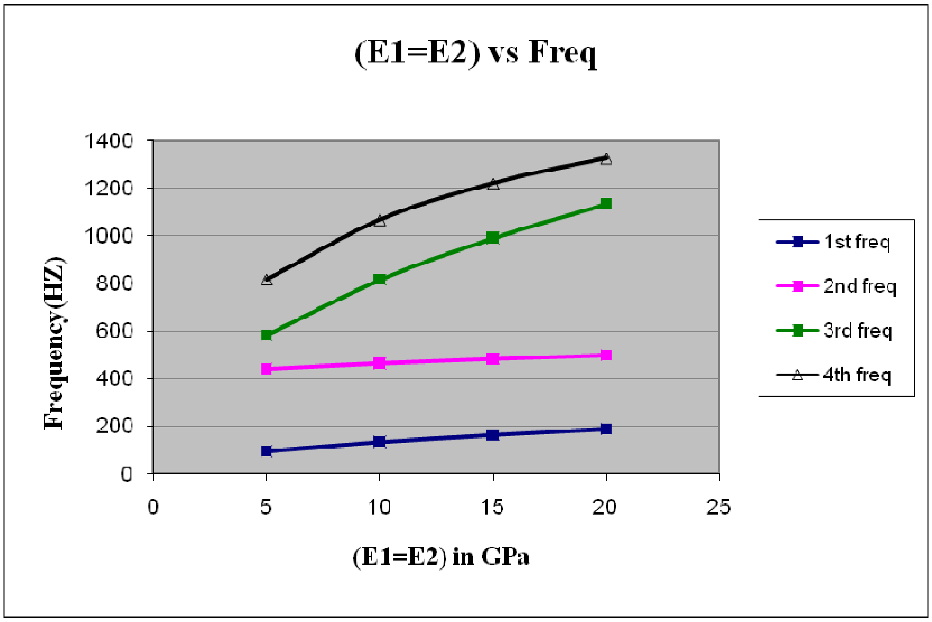


Fig 5.15:

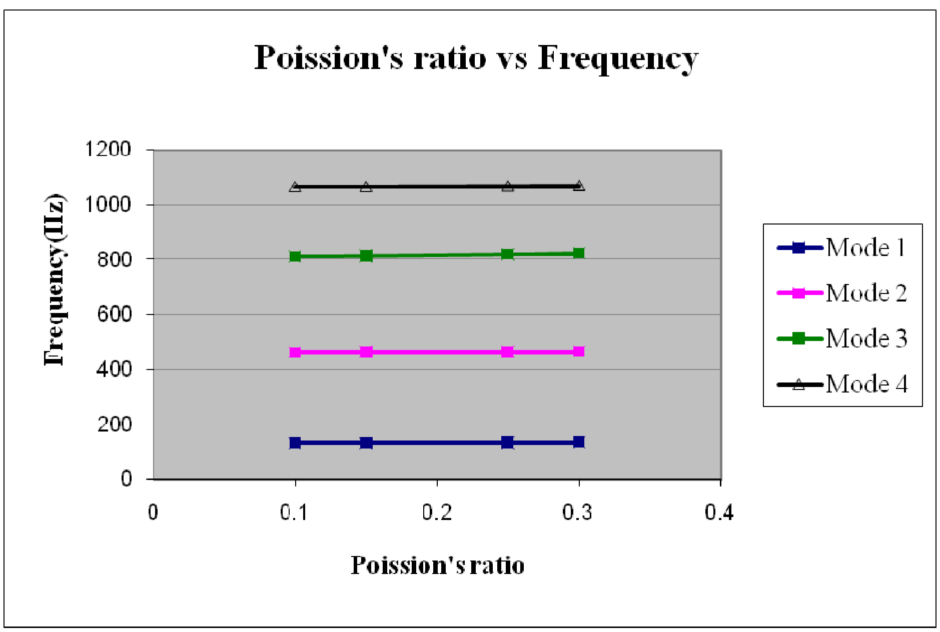


Fig 5.16:

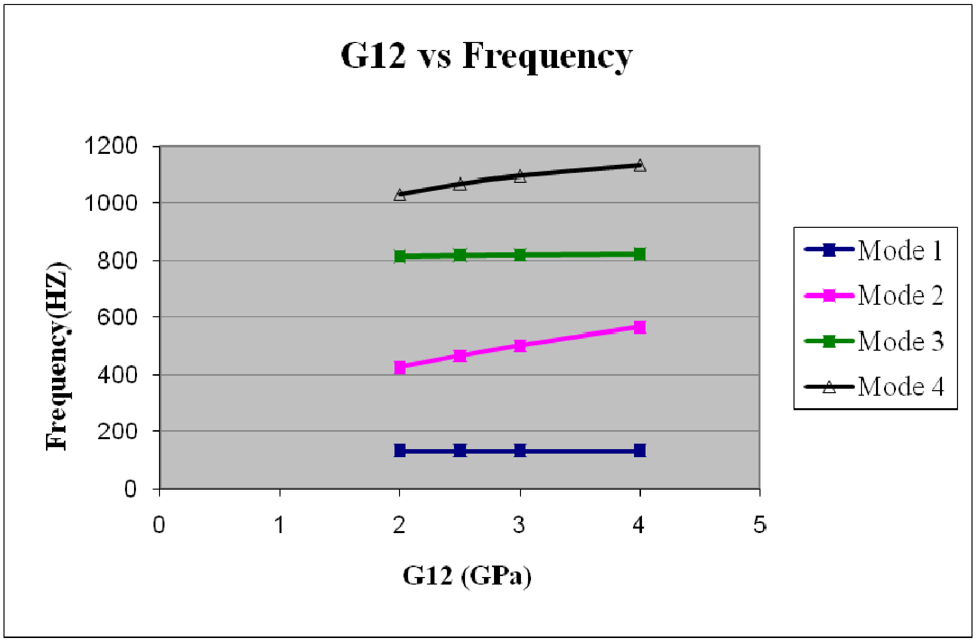


Fig 5.17:

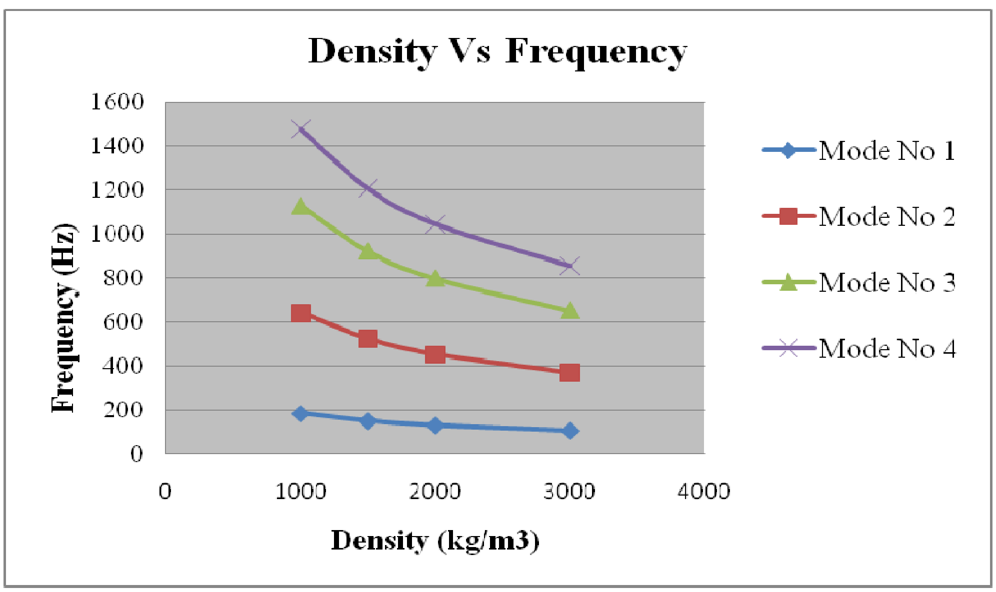
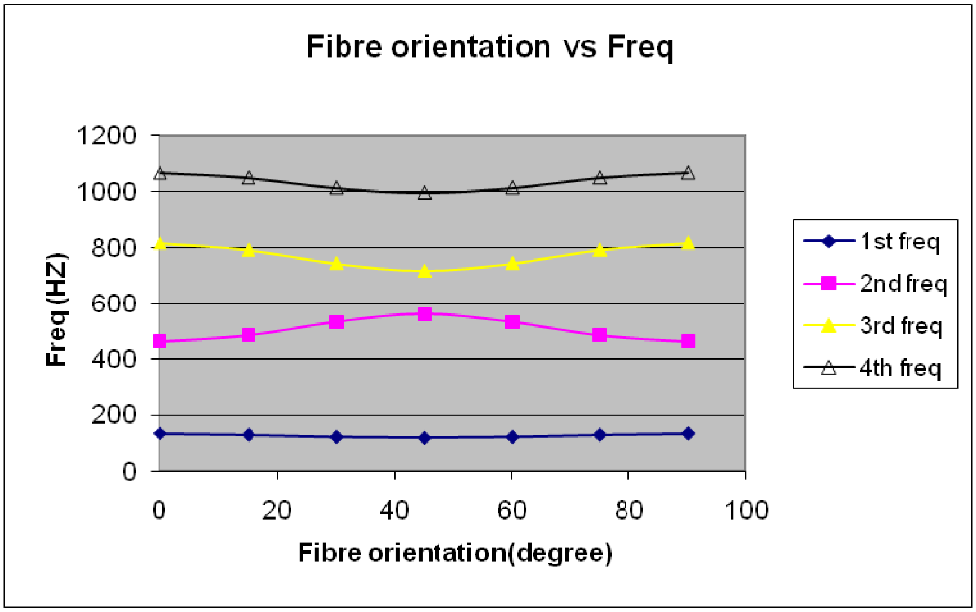


Fig 5.18:



Variation of natural frequencies with different parameters a/b ratio, b/h ratio, elastic modulus, shear modulus, poison's ratio, density, ply orientation for the 12-layered woven fiber Glass/Epoxy cantilever composite plate is done. Figures are plotted between different parameters and frequency.

As a/b ratio increases from 0.5 to 3, the natural frequencies decreases (Fig 5.12). Since, a/b ratio increases, the stiffness factor of the plate decreases. Hence, the natural frequency of the plate decreases. It decreases in parabolic shape for 1^{st} and 2^{nd} mode of frequency. However, it is less parabolic shape in 4^{th} mode of frequency. In case of 3^{rd} mode of frequency, natural frequency increases first and then decreases. The 3^{rd} mode of frequency of square plate (a/b=1) increases rapidly with respect to other a/b ratio.

Fig 5.13 shows that the natural modes of frequency decreases as the b/h value increases. Since, the width remains constant (a/b=2) the value of thickness decreases. So, the stiffness of the plate decreases and the natural frequencies decreases. It decreases in a parabolic shape. First decreases steeply, then it varies slowly.

From Fig 5.14, it is shown that natural frequency increases with the increasing value of elastic modulus. When the elastic modulus will increase, the stress will also increase. As the stress increases, the stiffness of the material increases. Hence, the natural frequency increases. It increases in a parabolic shape shown in the graph. In 1^{st} mode and 2^{nd} mode of frequency, frequency changes very slowly like flat shape of the figure. But it shows very good in case of 3^{rd} and 4^{th} mode of frequency.

Poisson's ratio is comparatively insensitive to the natural frequencies of woven fiber glass/epoxy composite plate. That is shown in Fig 5.15 very properly. It increases very slowly that is in decimal position at 1^{st} and 2^{nd} modes. At higher modes it varies slight greater than that of 1^{st} and 2^{nd} mode.

Fig 5.16 shows that natural frequency increases with increase of shear modulus value. Since the shear modulus increases, the shear force increases. Hence, the resistance of plate for shear force also increases. So, the stiffness factor increases and natural frequency increases. The 1^{st} and 3rd mode of frequency shows that there is slow increasing rate of natural frequency. The increasing rate decreases at higher value of shear modulus.

From Fig 5.17, it is studied that as the density increases the natural frequency decreases. As density increases, the mass must increase. Since, the mass increases, then the natural frequency definitely decrease. It is parabolic shape in decreasing order of natural frequency. The rate of decrease of natural frequency decreases as density value increases. As compared to the 1^{st} mode of frequency, other curves are more parabolic shape.

When angle of ply changes from 0^0 to 45^0 the natural frequency decreases and then increases up to 90^0 (Fig 5.18). The natural frequency value for 0^0, 90^0 and 15^0, 75^0 and 30^0, 60^0 is same. The lowest value of natural frequency is in the case of 45^0 ply orientation value. However, it increases up to 45^0 in 2^{nd} mode of frequency and then decreases.

5.5 PARAMETRIC STUDY FOR ALL EDGE CLAMPED PLATE

Table 5.14: Variation of natural frequencies with 'a/b' ratio for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b=breadth, h=thickness, b/h=10, $E_1=E_2=10.0GPa$, $G_{12}=G_{13}=2.5Gpa$, $\nu_{12}=0.20$, $\rho=1914kg/m^3$

a/b ratio	Mode No 1	Mode No 2	Mode No 3	Mode No 4
0.5	2702.4821	3354.8748	4480.7162	5677.9697
1	2342.6985	4408.9548	4408.9548	6029.2078
2	3340.1252	4178.6998	5739.9519	7939.9079
3	4479.2545	5207.0984	6065.6628	7429.6640

Table 5.15: Variation of natural frequencies with 'b/h' ratio for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, $E_1=E_2=10.0GPa$, $G_{12}=G_{13}=2.5Gpa$, $\nu_{12}=0.20$, $\rho=1914kg/m^3$

b/h ratio	Mode No 1	Mode No 2	Mode No 3	Mode No 4
10	3340.1252	4178.6998	5739.9519	7939.9079
25	1451.3058	1828.9316	2556.1441	3635.4426
50	735.4439	928.8148	1303.5503	1865.7607
100	369.2443	467.6663	659.0963	947.8286

Table 5.16: Variation of natural frequencies with 'E_1' value for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b/h=10, b=breadth, h=thickness, a/b=2, b/h=10, G_{12}=G_{13}=2.5Gpa, ν_{12}=0.20, ρ=1914kg/m^3

E_1 (GPa)	Mode No 1	Mode No 2	Mode No 3	Mode No 4
5	2532.3198	3275.8047	4543.7113	6179.5398
10	3340.1252	4178.6998	5739.9519	7939.9079
15	3896.3711	4821.0753	6582.6335	9029.8192
20	4317.9018	5317.2957	7226.7067	9781.0378

Table 5.17: Variation of natural frequencies with 'ν_{12}'value for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, E_1=E_2=10.0GPa, G_{12}=G_{13}=2.5Gpa, ρ=1914kg/m^3

ν_{12}	Mode No 1	Mode No 2	Mode No 3	Mode No 4
0.10	3283.4734	4085.7394	5612.9669	7785.5736
0.15	3308.2993	4128.3037	5671.3665	7855.9302
0.20	3340.1252	4178.6998	5739.9519	7939.9079
0.30	3427.1178	4306.0670	5911.6393	8109.1886

Table 5.18: Variation of natural frequencies with 'G_{12}' for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, E_1=E_2=10.0GPa, ν_{12}=0.20, ρ=1914kg/m^3

G_{12} (GPa)	Mode No 1	Mode No 2	Mode No 3	Mode No 4
2.0	3255.5034	4045.4021	5538.8193	7639.5714
2.5	3340.1252	4178.6998	5739.9519	7939.9079
3.0	3405.4850	4289.8788	5907.9165	8176.5597
4.0	3504.7188	4475.2168	6187.9913	8506.7226

Table 5.19: Variation of natural frequencies with 'ρ' (density) for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, E_1=E_2=10.0GPa, G_{12}=G_{13}=2.5Gpa, ν_{12}=0.20

ρ (kg/m^3)	Mode No 1	Mode No 2	Mode No 3	Mode No 4
1000	4620.9759	5781.1220	7941.0736	10984.6553
1500	3773.0110	4720.2664	6483.8594	8968.9335
2000	3267.5234	4087.8706	5615.1870	7767.3242
3000	2667.9217	3337.7323	4584.7810	6341.9937

Table 5.20: Variation of natural frequencies with 'θ' (angle) value for a 12-layered Glass/Epoxy all edge clamped composite plate;

a=length=0.014m, b=breadth, h=thickness, a/b=2, b/h=10, $E_1=E_2=10.0GPa$, $G_{12}=G_{13}=2.5Gpa$, $\nu_{12}=0.20$, $\rho=1914kg/m^3$

θ (degree)	Mode No 1	Mode No 2	Mode No 3	Mode No 4
0	3340.1252	4178.6998	5739.9519	7939.9079
15	3307.2783	4177.9359	5744.9960	7857.2749
30	3241.4742	4177.1755	5754.5825	7680.3807
45	3208.5033	4177.2603	5759.1270	7591.5639
60	3241.4745	4177.1757	5754.5829	7680.3807
75	3307.2783	4177.9359	5744.9960	7857.2749
90	3340.1252	4178.6998	5739.9519	7939.9079

Fig 5.19:

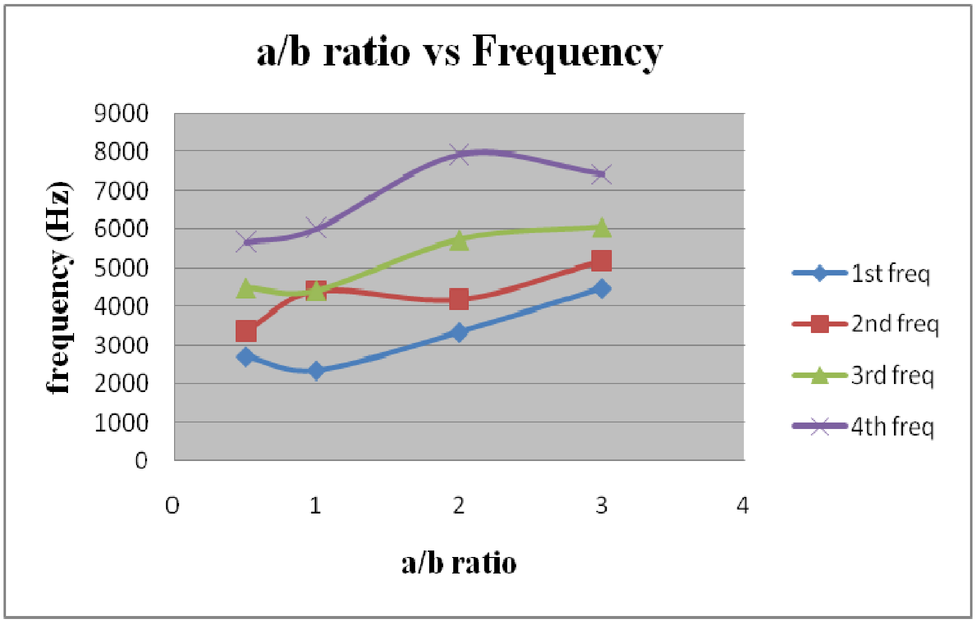


Fig 5.20:

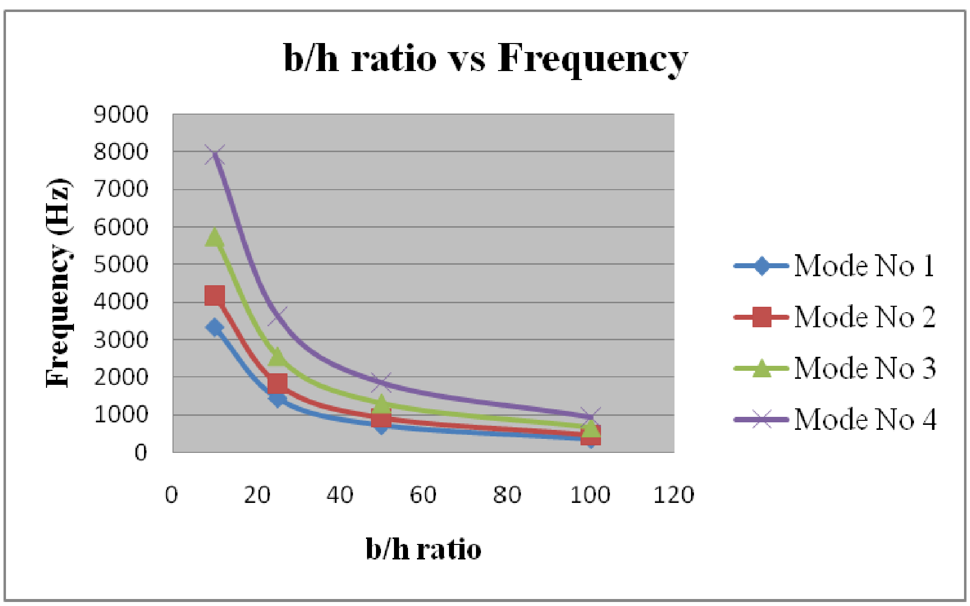


Fig 5.21:

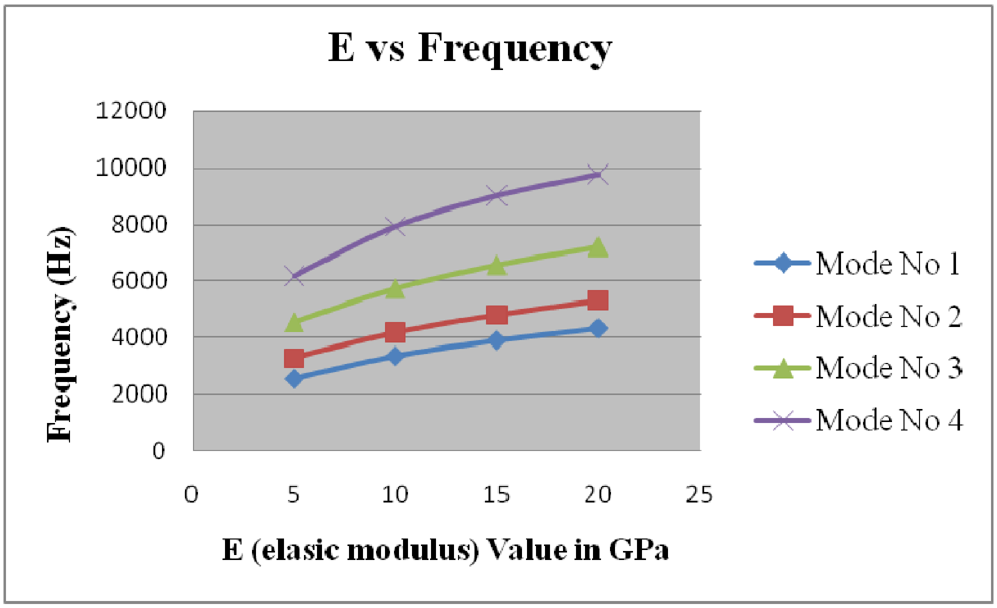


Fig 5.22:

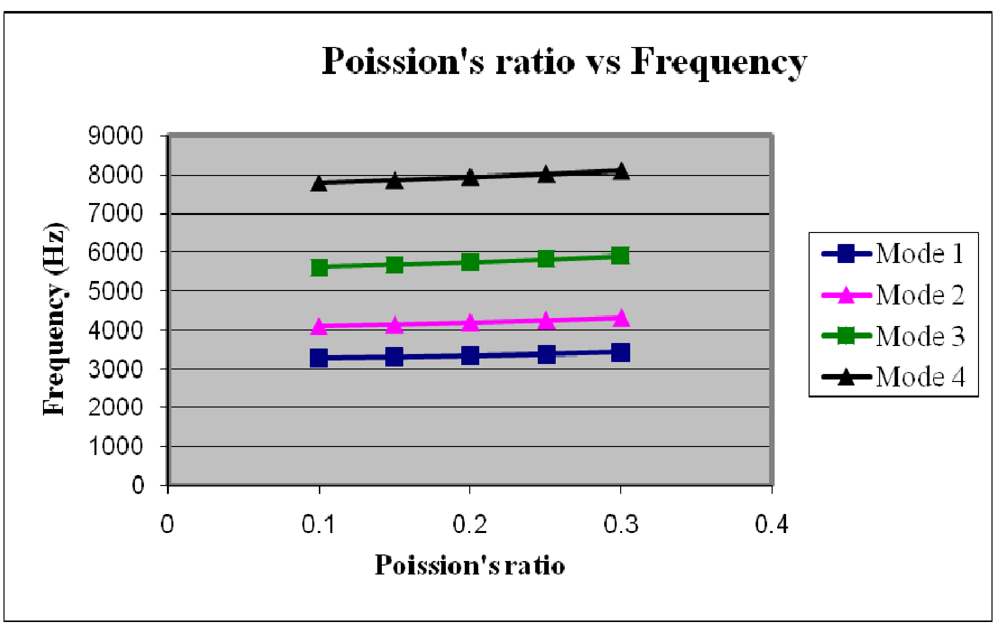


Fig 5.23:

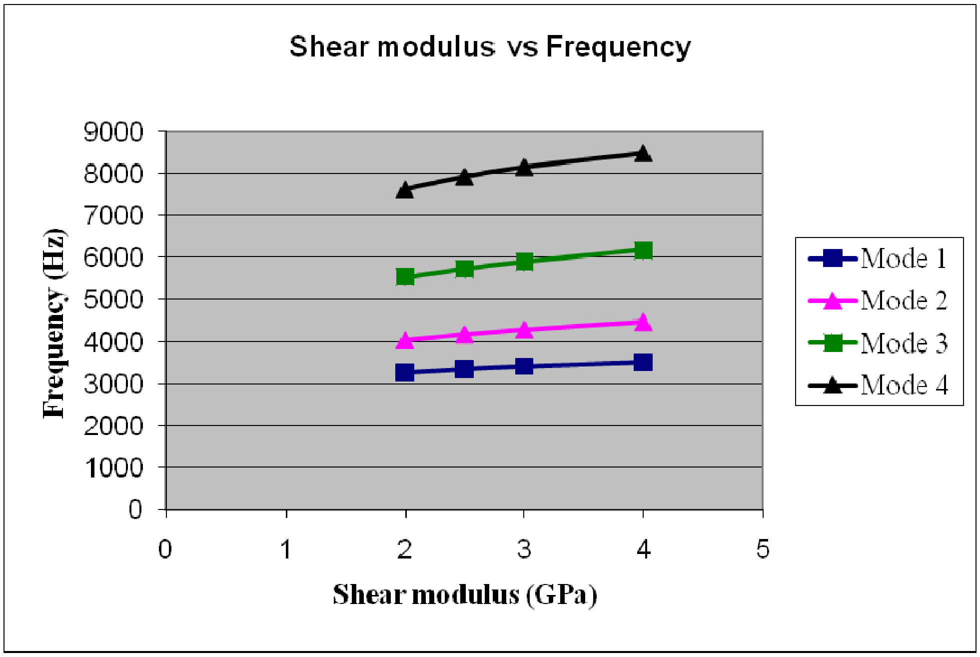


Fig 5.24:

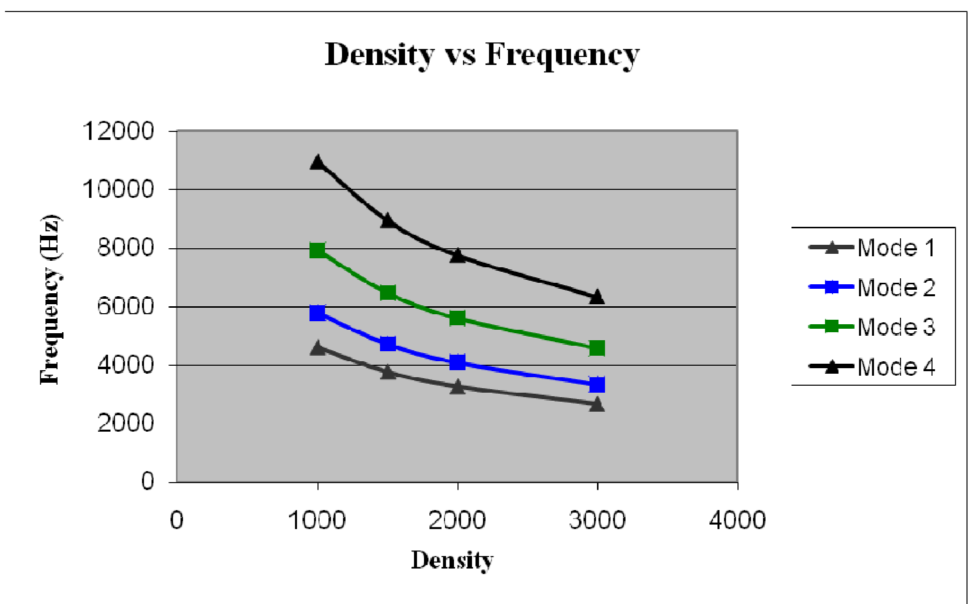
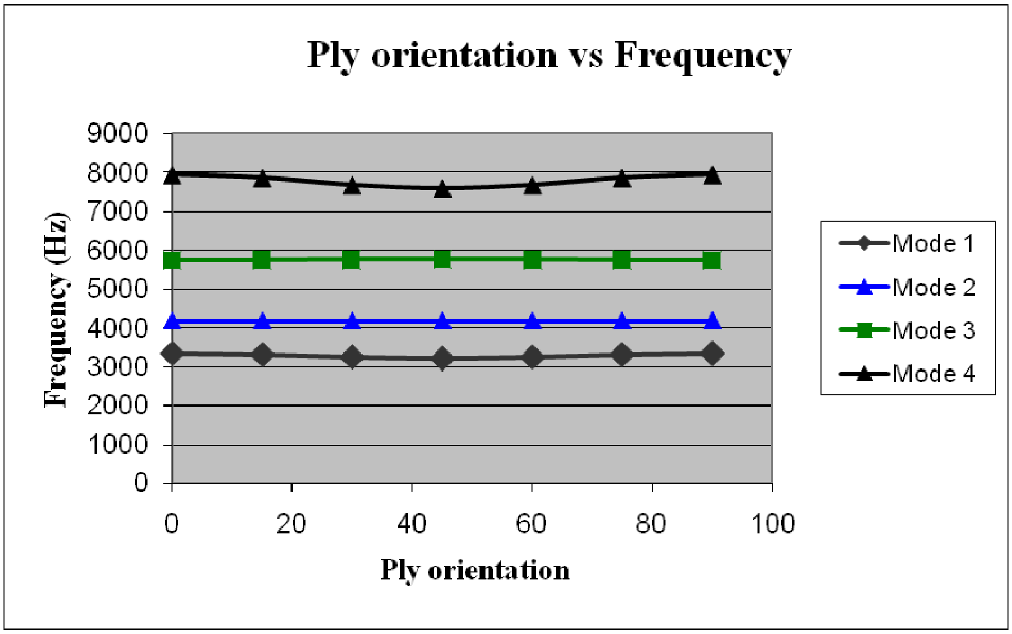


Fig 5.25:



As shown in the Fig 5.19, the value of natural frequency first decreases, then increases for 1st and 3rd mode of frequency. In case of 2nd frequency, 3rd frequency and 4th frequency it is like S-curve.

In Fig 5.20, natural frequency decreases with increase of b/h ratio in a parabolic curve shape. Like in the case of cantilever plate thickness decreases and stiffness factor decreases, for all edge clamped plate also stiffness decreases. But it is better curve than that of cantilever plate. Hence, natural frequency decreases.

From Fig 5.21, it is shown that all curves are in increasing order of natural frequency. It gives a parabolic curve. As the elastic modulus increases the flexural rigidity increases. So, stiffness increases and hence, natural frequency value increases. At higher mode, it increases more than lower mode. As the stiffness of all edge clamped plate is more than cantilever plate, it has a greater value of natural frequency. All the curves are shown as alike. In cantilever case two graphs have insignificant effect to increase of elastic modulus.

For increasing value of Poisson's ratio, natural frequency has insignificant change as shown in Fig 5.22. However, natural frequency increases with increase of Poisson's ratio. But the rate of increase is more than cantilever plate, due to higher stiffness factor.

Natural frequency varies in increasing rate with increasing of shear modulus value (Fig 5.23). It has better effect than cantilever plate. All the curves are in increasing order of parabolic shape.

From Fig 5.24, it is shown that density has same effect to frequency of all edge clamped plate as in case of cantilevered plate. However, it is better parabolic curve than that.

When angle of ply changes from 0^0 to 45^0 the natural frequency decreases and then increases up to 90^0 (5.25). The natural frequency value for 0^0, 90^0 and 15^0, 75^0 and 30^0, 60^0 is same. The lowest value of natural frequency is in the case of 45^0 ply orientation value. However, it increases up to 45^0 in 2^{nd} mode of frequency and then decreases. As compared to cantilever plate, variation of natural frequency up to 45^0 is in very slow decreasing order. In case of 3rd mode of frequency it increases, whereas it is 2^{nd} mode in cantilever plate.

Chapter 6

CONCLUSION

CONCLUSION

The natural frequencies of two varieties of cantilever laminated plates have been reported. The program result shows in general a good agreement with the existing literature. The experimental frequency data is in fair agreement with the program computation. The Percentage of error between experimental value and ANSYS package is within 15%. The difference is probably due to uncertainty in elastic properties and other described reasons.

From different boundary condition (cantilever and all edge fixed), it is found that the natural frequency of all edge fixed plate is very higher than cantilever plate. Program results show clearly that changes in elastic properties yield to different dynamic behavior of the plates. Natural frequency decreases with the increasing of a/b ratio, b/h ratio, and density value. Whereas, it increases with the increase of shear modulus, and elastic modulus. It increases slightly with Poisson's ratio variation; but it is comparatively insensitive.

Natural frequency decreases as the ply orientation increases up to 45^0 and again increases up to 90^0. The lowest value is at 45^0 ply orientation.

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APPENDIX

INTRODUCTION

The Fourier transform is a method for representing a time history signal in terms of a frequency domain function. Specifically, the Fourier transform represents a signal in terms of its spectral components. The fourier transform is a complex exponential transform which is related to the Laplace transform. The Fourier transform is also referred to as a trigonometric transformation since the complex exponential function can be represented in terms of trigonometric functions.

The Fourier transform is often applied to digital time histories. The time histories are sampled from measured analog data. The transform calculation method, however requires a relatively high number of mathematical operations. As an alternative, a Fast Fourier Transform (FFT) method has been developed to simplify this calculation. The purpose of this tutorial is to present a Fast Fourier transform algorithm.

FOURIER TRANSFORM THEORY

(Fast Fourier Transform) A class of algorithms used in digital signal processing that break down complex signals into elementary components.

A Fast Fourier Transform (FFT) is an efficient algorithm to compute the Discrete Fourier Transform (DFT) and its inverse. FFTs are of great importance to a wide variety of applications, from digital signal processing to solving partial differential equations to algorithms for quickly multiplying large integers. This article describes the algorithms, of which there are many; see Discrete Fourier Transform for properties and applications of the transform.